

■ Use mathematical induction to prove the following.

[1] $1 + 4 + 9 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}, n \in \mathbb{Z}$

[2] Where F_1, F_2 , etc. are the Fibonacci numbers, prove

$$F_1 + F_2 + \dots + F_n = F_{n+2} - 1$$

[3] $(2n)! < (n!)^2 4^{n-1}$ for $\forall n \in \mathbb{Z}, n \geq 5$

[4] $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \dots + \frac{1}{(n-1)n} = \frac{n-1}{n}$ for $n = 2, 3, 4, \dots$

[5] $6 \mid (n^3 + 5n)$ for $\forall n \in \mathbb{Z}^+$

[6] $\frac{1 \cdot 3 \cdot 5 \cdot \dots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n)} \geq \frac{1}{2n}, n \in \mathbb{Z}^+$

[7] $\frac{(4n-2)!}{8^n} = \text{an integer, for } n = 5, 6, 7, \dots$

[8] Let $n \in \mathbb{Z}^+$ and A_1, A_2, \dots, A_n subsets of \mathbb{U} .

$$\text{Prove } \overline{A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n} = \overline{A_1} \cup \overline{A_2} \cup \dots \cup \overline{A_n}$$

[9] $2 \cdot 6 \cdot 10 \cdot 14 \cdot \dots \cdot (4n-2) = \frac{(2n)!}{n!}$

[10] (Bernoulli's Inequality)

Let $a > -1$

Thm $(1+a)^n \geq 1+na$, for $n = 0, 1, 2, \dots, a \in \text{constant}$

[11] $1 \cdot 3 \cdot 5 \cdot 7 \cdot \dots \cdot (2n-1) \geq 2 \cdot 4 \cdot 6 \cdot \dots \cdot (2n-2), n \geq 2, n \in \mathbb{Z}$

[12] $F_n \geq \left(\frac{5}{4}\right)^n, n \geq 3, F_n = n \text{ th Fibonacci number}$

[13] $9 \mid (4^n + 5^n)$, $n \in \text{odd}$

[14] Definition. If a is a real number, then $a^1 = a$ and, for every positive integer k , $a^{k+1} = a^k \cdot a$

Prove: $a^m \cdot a^n = a^{m+n}$

[15] (DeMoivre Thm)

If $z = |z|(\cos \phi + i \sin \phi)$, then $z^n = |z|^n (\cos n\phi + i \sin n\phi)$ for each natural number n .

[16] $a \equiv b \pmod{m} \implies a^k \equiv b^k \pmod{m}$ for each positive integer k .